

Hamiltonian ADM Gravity in Non-Harmonic Gauges with Well Defined Non-Euclidean 3-Spaces: How Much Darkness can be Explained as a Relativistic Inertial Effect?

Luca Lusanna

*Sezione INFN di Firenze
Polo Scientifico
Via Sansone 1
50019 Sesto Fiorentino (FI), Italy
Phone: 0039-055-4572334
FAX: 0039-055-4572364
E-mail: lusanna@fi.infn.it*

Abstract

In special and general relativity the synchronization convention of distant clocks may be simulated with a mathematical definition of global non-inertial frames (the only ones existing in general relativity due to the equivalence principle) with well-defined instantaneous 3-spaces. For asymptotically Minkowskian Einstein space-times this procedure can be used at the Hamiltonian level in the York canonical basis, where it is possible for the first time to disentangle tidal gravitational degrees of freedom from gauge inertial ones. The most important inertial effect connected with clock synchronization is the York time ${}^3K(\tau, \sigma^r)$, not existing in Newton gravity. This fact opens the possibility to describe some aspects of *darkness* as a relativistic inertial effect in Einstein gravity by means of a Post-Minkowskian reformulation of the Celestial Reference System ICRS.

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In classical and quantum physics predictability is possible only if the relevant partial differential equations have a well-posed Cauchy problem, whose pre-requisite is the existence of a well defined 3-space (i.e. a clock synchronization convention) supporting the Cauchy data.

In Galilei space-time there is no problem: time and Euclidean 3-space are absolute.

Instead there is no intrinsic notion of 3-space, simultaneity, 1-way velocity of light (two distant clocks are involved) in the absolute Minkowski space-time: only the light-cone is intrinsically given as the locus of incoming and outgoing radiation. The light postulate says that the 2-way (only one clock is involved) velocity of light c is isotropic and constant. Its codified value replaces the rods (i.e. the standard of length) in modern metrology, where an atomic clock gives the standard of time. Einstein's 1/2 synchronization convention¹ selects the Euclidean 3-spaces $x^o = ct = \text{const.}$ of the inertial frames centered on inertial observers: only in this case the 2-way and 1-way light velocities coincide. However, with realistic accelerated observers the convention breaks down and till recently there was no definition of global non-inertial 3-spaces due to the coordinate singularities present in the *1+3 point of view* (only the world-line of a time-like observer is given) both with Fermi coordinates (crossing of the 3-spaces) and rotating frames (the horizon problem of the rotating disk).

In Ref.[1] the theory of global non-inertial frames is fully developed in the *3+1 point of view*: besides the observer world-line one gives an admissible 3+1 splitting of Minkowski space-time, i.e. a nice foliation whose leaves are instantaneous 3-spaces. Lorentz-scalar observer-dependent radar 4-coordinates $\sigma^A = (\tau; \sigma^r)$ are used: τ is an arbitrary increasing function of the observer proper time and σ^r are curvilinear 3-coordinates on the 3-spaces Σ_τ with the observer as origin. Each 3-space is asymptotically Euclidean with asymptotic inertial observers at spatial infinity. The inverse transformation $\sigma^A \mapsto x^\mu = z^\mu(\tau, \sigma^r)$ defines the embeddings of the 3-spaces Σ_τ into Minkowski space-time and the induced 4-metric is $g_{AB}[z(\tau, \sigma^r)] = [z_A^\mu \eta_{\mu\nu} z_B^\nu](\tau, \sigma^r)$, where $z_A^\mu = \partial z^\mu / \partial \sigma^A$ and ${}^4\eta_{\mu\nu} = \epsilon(+ - - -)$ is the flat metric ($\epsilon = \pm 1$ according to either the particle physics $\epsilon = 1$ or the general relativity $\epsilon = -1$ convention). While the 4-vectors $z_r^\mu(\tau, \sigma^u)$ are tangent to Σ_τ , so that the unit normal $l^\mu(\tau, \sigma^u)$ is proportional to $\epsilon^\mu{}_{\alpha\beta\gamma} [z_1^\alpha z_2^\beta z_3^\gamma](\tau, \sigma^u)$, we have $z_r^\mu(\tau, \sigma^r) = [N l^\mu + N^r z_r^\mu](\tau, \sigma^r)$ ($N(\tau, \sigma^r) = \epsilon [z_\tau^\mu l_\mu](\tau, \sigma^r)$ and $N_r(\tau, \sigma^r) = -\epsilon g_{\tau r}(\tau, \sigma^r)$ are the lapse and shift functions).

The foliation is nice and admissible if it satisfies the conditions:

- 1) $N(\tau, \sigma^r) > 0$ in every point of Σ_τ (the 3-spaces never intersect, avoiding the coordinate singularity of Fermi coordinates);
- 2) $\epsilon {}^4g_{\tau\tau}(\tau, \sigma^r) > 0$, so to avoid the coordinate singularity of the rotating disk, and with the positive-definite 3-metric ${}^3g_{rs}(\tau, \sigma^u) = -\epsilon {}^4g_{rs}(\tau, \sigma^u)$ having three positive eigenvalues (these are the Møller conditions [1]);
- 3) all the 3-spaces Σ_τ must tend to the same space-like hyper-plane at spatial infinity (so that there are always asymptotic inertial observers to be identified with the fixed stars).

These conditions imply that global *rigid* rotations are forbidden in relativistic theories. In Ref.[1] there is the expression of the admissible embedding corresponding to a 3+1 splitting

¹ An inertial observer A send a ray of light at x_i^o towards the (in general accelerated) observer B; the ray is reflected towards A at a point P of B world-line and then reabsorbed by A at x_f^o ; by convention P is synchronous with the mid-point between emission and absorption on A's world-line, i.e. $x_P^o = x_i^o + \frac{1}{2}(x_f^o - x_i^o)$.

of Minkowski space-time with parallel space-like hyper-planes (not equally spaced due to a linear acceleration) carrying differentially rotating 3-coordinates without the coordinate singularity of the rotating disk. It is the first consistent global non-inertial frame of this type.

As shown in Refs.[1, 2] every isolated system (particles, strings, fluids, fields) admitting a Lagrangian $\mathcal{L}(\text{matter})$ can be reformulated as a *parametrized Minkowski theory*, in which the new embedding-dependent Lagrangian is $\mathcal{L}(\text{matter}, g_{AB}[z])$. This action is invariant under frame-preserving 4-diffeomorphisms, so that the embeddings are *gauge variables* and the ten components of $g_{AB}[z]$ are the special-relativistic *inertial potentials*². A change of clock synchronization (of the shape of Σ_τ) and/or of the 3-coordinates into the 3-spaces is a gauge transformation: physics does not change, only the appearances of phenomena change.

In this formulation the description of matter has to be done with quantities which know the instantaneous 3-spaces Σ_τ . For instance a Klein-Gordon field $\tilde{\phi}(x)$ will be replaced with $\phi(\tau, \sigma^r) = \tilde{\phi}(z(\tau, \sigma^r))$; the same for every other field. Instead for a relativistic particle with world-line $x^\mu(\tau)$ we must make a choice of its energy sign: then it will be described by 3-coordinates $\eta^r(\tau)$ defined by the intersection of the world-line with Σ_τ : $x^\mu(\tau) = z^\mu(\tau, \eta^r(\tau))$. Differently from all the previous approaches to relativistic mechanics, the dynamical configuration variables are the 3-coordinates $\eta_i^r(\tau)$ and not the world-lines $x_i^\mu(\tau)$ (to rebuild them in an arbitrary frame we need the embedding defining that frame!).

The *inertial rest-frame instant form* of the isolated system [1, 2] is obtained by restricting the embedding to the inertial rest-frame centered on the Fokker-Pryce center of inertia: its Euclidean Wigner-covariant 3-spaces are orthogonal to the conserved 4-momentum of the isolated system. Every isolated system can be described as a decoupled non-local (and therefore *un-observable*) canonical non-covariant Newton-Wigner external center of mass³, with an associated external realization of the Poincare' algebra, carrying a pole-dipole structure: the invariant mass M and the rest spin \vec{S} of the isolated system. By construction, they depend upon Wigner-covariant relative variables describing the internal dynamics of the isolated system⁴.

The world-lines $x_i^\mu(\tau)$ of the particles are derived (interaction-dependent) quantities and in general they do not satisfy vanishing Poisson brackets: already at the classical level a non-commutative structure emerges!

The definition of relativistic atomic physics (scalar positive-energy charged particles plus

² They generate the relativistic apparent forces in the non-inertial frame and in the non-relativistic limit they reduce to the Newtonian inertial potentials. The extrinsic curvature ${}^3K_{rs}(\tau, \sigma^u) = [\frac{1}{2N} (N_{r|s} + N_{s|r} - \partial_\tau {}^3g_{rs})](\tau, \sigma^u)$, describing the *shape* of the instantaneous 3-spaces of the non-inertial frame as embedded 3-manifolds of Minkowski space-time, is a functional of the independent inertial potentials⁴ g_{AB} .

³ It is convenient to replace it with its initial value, namely with the Jacobi data of the Hamilton-Jacobi formulation.

⁴ Inside the Wigner 3-spaces there is an unfaithful internal realization of the Poincare' algebra, determined by the energy-momentum tensor, whose energy is the invariant mass and whose angular momentum is the rest spin. The internal 3-momentum vanishes being the rest-frame condition. The internal center of mass inside the Wigner 3-spaces is eliminated by the vanishing of the internal (interaction-dependent) Lorentz boosts, avoiding a double counting of this collective variable.

the electro-magnetic field in the radiation gauge with Grassmann-valued electric charges to regularize self-energies) and of its Poincare' generators becomes possible [3–5] in this framework. The identification of the Darwin potential, to be added to the Coulomb one, in this classical setting establishes a contact with the theory of relativistic bound states, whose constituents must be synchronized (absence of relative times).

Also a new formulation of *relativistic quantum mechanics and entanglement* was given [6]. The use of the static Jacobi data for the external center of mass avoids the causality problems connected with the instantaneous spreading of wave packets. Due to the need of clock synchronization for the definition of the instantaneous 3-spaces, the Hilbert space $H = H_{com,HJ} \otimes H_{rel}$ ($H_{com,HJ}$ is the Hilbert space of the external center of mass in the Hamilton-Jacobi formulation, while H_{rel} is the Hilbert space of the internal relative variables) is not unitarily equivalent to $H_1 \otimes H_2 \otimes \dots$, where H_i are the Hilbert spaces of the individual particles. As a consequence, at the relativistic level the zeroth postulate of non-relativistic quantum mechanics does not hold: the Hilbert space of composite systems is not the tensor product of the Hilbert spaces of the sub-systems. The non validity of the zeroth postulate and the *non-locality* of Poincare' generators imply a *kinematical non-locality* and a *kinematical spatial non-separability* introduced by special relativity, which reduce the relevance of *quantum non-locality* in the study of the foundational problems of quantum mechanics which have to be rephrased in terms of relative variables.

The replacement of clock synchronization with an admissible 3+1 splitting can be used also in general relativity (GR), where also the space-time becomes dynamical [7], being determined by Einstein equations modulo 4-coordinate transformations (the gauge group of GR). We will define global non-inertial frames (the only ones existing in the large in GR due to the equivalence principle) with admissible 3+1 splittings and radar 4-coordinates in globally hyperbolic, asymptotically Minkowskian space-times in the framework of ADM canonical gravity. With suitable boundary conditions, eliminating super-translations [8], the asymptotic symmetries reduce to the ADM Poincare' group ⁵ and the non-Euclidean 3-spaces are orthogonal to the conserved ADM 4-momentum at spatial infinity [9]: this is a *non-inertial rest frame* of the 3-universe (see Ref.[1] for the non-inertial rest-frame instant form in special relativity). There are asymptotic inertial observers with spatial axes identified by means of the fixed stars of star catalogues.

As a consequence, the 3-universe (the isolated system "gravitational field plus matter") can be described as a decoupled non-covariant non-observable external pseudo-particle carrying a pole-dipole structure, whose mass and spin are identified by the ADM weak energy and by the ADM angular momentum. Instead the ADM 3-momentum vanishes, since this determines the rest-frame condition. The vanishing of the ADM Lorentz boosts eliminate the internal center of mass of the 3-universe.

In absence of matter Christodoulou - Klainermann space-times [10] are compatible with this description.

⁵ For $G = 0$ it reduces to the Poincare' group of the matter in Minkowski non-inertial frames. In this way, after a restriction to inertial frames we can recover all the results of the standard model of elementary particles, which are connected with properties of the representations of the Poincare' group in inertial frames of Minkowski space-time.

Now the dynamical variable is not the embedding but the 4-metric, which determines the dynamical chrono-geometrical structure of space-time by means of the line element: it teaches to massless particles which are the allowed trajectories in each point ⁶. Since tetrad gravity is more natural for the coupling of gravity to the fermions, the 4-metric is decomposed in terms of cotetrads, ${}^4g_{AB} = E_A^{(\alpha)} {}^4\eta_{(\alpha)(\beta)} E_B^{(\beta)}$ ⁷, and the ADM action, now a functional of the 16 fields $E_A^{(\alpha)}(\tau, \sigma^r)$, is taken as the action for ADM tetrad gravity [9]. This leads to an interpretation of gravity based on a congruence of time-like observers endowed with orthonormal tetrads: in each point of space-time the time-like axis is the unit 4-velocity of the observer, while the spatial axes are a (gauge) convention for observer's gyroscopes.

In canonical ADM tetrad gravity there are 16 fields, 16 conjugate momenta, 14 first-class constraints, generators of Hamiltonian gauge transformations, 14 gauge variables, the *GR inertial effects* and 2+2 physical variables, the *tidal effects* (the gravitational waves after linearization). As shown in Refs.[9, 12], in our family of space-times the Dirac Hamiltonian turns out to be the *weak* ADM energy ⁸ plus constraints. Therefore in this family of space-times there is *not a frozen picture*, like in the family of spatially compact without boundary space-times considered in loop quantum gravity, where the Dirac Hamiltonian is a combination of constraints.

In Ref.[9] a York canonical basis, adapted to ten first-class constraints, was identified: this allows for the first time to get the explicit identification of the inertial and tidal variables. It implements the York map of Ref.[13] and diagonalizes the York-Lichnerowicz approach [14]. Its final form is $(\alpha_{(a)}(\tau, \sigma^r))$ are angles, ${}^3e_{(a)r}(\tau, \sigma^r)$ are cotriads on the 3-space, $1 + n(\tau, \sigma^r)$ and $\bar{n}_{(a)}(\tau, \sigma^r)$ are the lapse and shift functions respectively)

⁶ In 2013 the ESA-ACES mission [11] on the synchronization of atomic clocks between Earth and the Space Station will make the first precision measurement of the gravitational redshift created by the geo-potential, i.e. of the $1/c^2$ modifications of the Minkowski light-cone. Every approach to quantum gravity will have to reproduce these data. A varying light-cone is a non-perturbative effect in every quantum field theory, string included, because to define the Fock space one needs the Fourier decomposition of fields on a fixed background space-time with a fixed light-cone. On the other hand in loop quantum gravity one has still to find a well defined coarse graining identifying Minkowski space-time and perturbations around it.

⁷ (α) are flat indices; the cotetrads $E_A^{(\alpha)}$ are the inverse of the tetrads $E_{(\alpha)}^A$ connected to the world tetrads by $E_{(\alpha)}^\mu(x) = z_A^\mu(\tau, \sigma^r) E_{(\alpha)}^A(z(\tau, \sigma^r))$.

⁸ It is a volume integral over 3-space of a coordinate-dependent energy density. It is weakly equal to the *strong* ADM energy, which is a flux through a 2-surface at spatial infinity.

$\varphi_{(a)}$	$\alpha_{(a)}$	n	$\bar{n}_{(a)}$	θ^r	$\tilde{\phi}$	$R_{\bar{a}}$
$\pi_{\varphi_{(a)}} \approx 0$	$\pi_{\alpha_{(a)}}^{(\alpha)} \approx 0$	$\pi_n \approx 0$	$\pi_{\bar{n}_{(a)}} \approx 0$	$\pi_r^{(\theta)}$	$\pi_{\tilde{\phi}} = \frac{c^3}{12\pi G} {}^3K$	$\Pi_{\bar{a}}$

$$\begin{aligned}
{}^3e_{(a)r} &= R_{(a)(b)}(\alpha_{(c)}) {}^3\bar{e}_{(b)r} = R_{(a)(b)}(\alpha_{(c)}) V_{rb}(\theta^i) \tilde{\phi}^{1/3} e^{\sum_{\bar{a}}^{1,2} \gamma_{\bar{a}a} R_{\bar{a}}}, \\
{}^4g_{\tau\tau} &= \epsilon [(1+n)^2 - \sum_a \bar{n}_{(a)}^2], \quad {}^4g_{\tau r} = -\epsilon \bar{n}_{(a)} {}^3\bar{e}_{(a)r}, \\
{}^4g_{rs} &= -\epsilon {}^3g_{rs} = -\epsilon \tilde{\phi}^{2/3} \sum_a V_{ra}(\theta^i) V_{sa}(\theta^i) e^{2 \sum_{\bar{a}}^{1,2} \gamma_{\bar{a}a} R_{\bar{a}}},
\end{aligned} \tag{0.1}$$

In this York canonical basis the *inertial effects* are described by the arbitrary gauge variables $\alpha_{(a)}$, $\varphi_{(a)}$, $1+n$, $\bar{n}_{(a)}$, θ^i , 3K , while the *tidal effects*, i.e. the physical degrees of freedom of the gravitational field, by the two canonical pairs $R_{\bar{a}}$, $\Pi_{\bar{a}}$, $\bar{a} = 1, 2$. The momenta $\pi_r^{(\theta)}$ and the 3-volume element $\tilde{\phi} = \sqrt{\det {}^3g_{rs}}$ have to be found as solutions of the super-momentum and super-hamiltonian (i.e. the Lichnerowicz equation) constraints, respectively.

The gauge variables $\alpha_{(a)}$, $\varphi_{(a)}$ parametrize the extra O(3,1) gauge freedom of the tetrads (the gauge freedom for each observer to choose three gyroscopes as spatial axes and to choose the law for their transport along the world-line). We have studied in detail the Schwinger time gauges where we impose the gauge fixings $\varphi_{(a)}(\tau, \sigma^r) \approx 0$, $\alpha_{(a)}(\tau, \sigma^r) \approx 0$ so that the tetrads become adapted to the 3+1 splitting (the time-like tetrad coincides with the unit normal to the 3-space).

The gauge angles θ^i (i.e. the director cosines of the tangents to the three coordinate lines in each point of Σ_τ) describe the freedom in the choice of the 3-coordinates σ^r on each 3-space: their fixation implies the determination of the shift gauge variables $\bar{n}_{(a)}$, namely the appearances of gravito-magnetism in the chosen 3-coordinate system.

One momentum is a gauge variable (a reflex of the Lorentz signature): the *York time*, i.e. the trace ${}^3K(\tau, \sigma^r)$ of the *extrinsic curvature* of the non-Euclidean 3-spaces as 3-sub-manifolds of space-time. This inertial effect (absent in Newtonian gravity with its absolute Euclidean 3-space) describes the GR remnant of the special-relativistic gauge freedom in clock synchronization. Its fixation determines the lapse function.

In the York canonical basis the Hamilton equations generated by the Dirac Hamiltonian $H_D = \hat{E}_{ADM} + (\text{constraints})$ are divided in four groups: A) four contracted Bianchi identities, namely the evolution equations for $\tilde{\phi}$ and $\pi_i^{(\theta)}$ (they say that given a solution of the constraints on a Cauchy surface, it remains a solution also at later times); B) four evolution equation for the four basic gauge variables θ^i and 3K : these equations determine the lapse and the shift functions once four gauge fixings for the basic gauge variables are added; C) four evolution equations for the tidal variables $R_{\bar{a}}$, $\Pi_{\bar{a}}$; D) the Hamilton equations for matter, when present.

Once a gauge is completely fixed, the Hamilton equations become deterministic. Given a solution of the super-momentum and super-Hamiltonian constraints and the Cauchy data for

the tidal variables on an initial 3-space, we can find a solution of Einstein's equations in radar 4-coordinates adapted to a time-like observer. To it there is associated a special 3+1 splitting of space-time with dynamically selected instantaneous 3-spaces in accord with Ref.[7]. Then we can get pass to adapted world 4-coordinates ($x^\mu = z^\mu(\tau, \sigma^r) = x_o^\mu + \epsilon_A^\mu \sigma^A$) and we can describe the solution in every 4-coordinate system by means of 4-diffeomorphisms.

In Ref.[15] we study the coupling of N charged scalar particles plus the electro-magnetic field to ADM tetrad gravity in this class of asymptotically Minkowskian space-times without super-translations. To regularize the self-energies both the electric charge and the sign of the energy of the particles are Grassmann-valued. The introduction of the non-covariant radiation gauge allows to reformulate the theory in terms of transverse electro-magnetic fields and to extract the generalization of the Coulomb interaction among the particles in the Riemannian instantaneous 3-spaces of global non-inertial frames.

From the Hamilton equations in the York canonical basis [15], followed by a Post-Minkowskian linearization with the asymptotic flat Minkowski 4-metric at spatial infinity as background, it has been possible to develop a theory of gravitational waves with asymptotic background propagating in the non-Euclidean 3-spaces Σ_τ of a family of *non-harmonic 3-orthogonal* gauges⁹ parametrized by the values of the York time ${}^3K(\tau, \sigma^r)$ (the left gauge freedom in the shape of Σ_τ).

The conceptual problem of the GR gauge freedom in the choice of the 4-coordinates is *solved at the experimental level inside the Solar system by the choice of a convention for the description of matter*: a) for satellites near the Earth (like the GPS ones) one uses NASA 4-coordinates compatible with the terrestrial ITFR2003 and geocentric GCRS IAU2000 [16] frames; b) for planets in the Solar System one uses the barycentric BCRS-IAU2000 [16] frame. These frames are compatible with "quasi-inertial frames" in Minkowski space-time. These are metrological choices like the choice of a certain atomic clock as standard of time.

In astronomy the positions of stars and galaxies are determined from the data (luminosity, light spectrum, angles) on the sky as living in a 4-dimensional nearly-Galilei space-time with the celestial ICRS [16] frame considered as a "quasi-inertial frame" (all galactic dynamics is Newtonian gravity), in accord with the standard FRW Λ CDM cosmological model when the constant intrinsic 3-curvature of 3-spaces is zero (as implied by the CMB data[17]). To reconcile all the data with this 4-dimensional reconstruction one must postulate the existence of dark matter and dark energy as the dominant components of the classical universe after the recombination 3-surface!

Our proposal is to define a *Post-Minkowskian ICRS* with non-Euclidean 3-spaces, whose intrinsic 3-curvature (due essentially to gravitational waves) is small, in such a way that the York time be (at least partially) fitted to the observational data implying the presence of dark matter. As shown in Ref.[15] the Post-Newtonian limit of the Post-Minkowskian Hamilton equations of particles in this family of gauges reproduces Kepler equations plus a v/c term depending on the York time (the arbitrary gauge function). Therefore there is the concrete possibility (under investigation) to explain the rotation curves of galaxies [18] as a *relativistic inertial effect inside Einstein GR* (choice of a York time compatible with observations [19]) without modifications: a) of Newton gravity like in MOND [20]; b) of GR

⁹ The 3-metric in Σ_τ is diagonal like in astronomical frames GCRS and BCRS.

like in $f(R)$ theories [21]; c) of particle physics with the introduction of WIMPS [22]. Then, the next step will be to study the dependence on the York time of quantities like redshift, luminosity distance, gravitational lensing... and to see which information on the York time can be extracted from the data supporting dark energy.

In conclusion the reformulation of clock synchronization as the existence of well-defined non-Euclidean 3-spaces with the gauge freedom of the York time plus the proposed way out from the GR gauge problem using the observational metrological conventions may help in reducing the dark side of the universe to a relativistic inertial effect inside Einstein GR by means of a Post-Minkowskian definition of ICRS, which will be also useful for the ESA-GAIA mission [23] (cartography of the Milky Way) and for the possible anomalies inside the Solar System [24].

Finally the transition to cosmology should be done with approaches of the type of back-reaction [25].

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